Домашка

① $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ -1 & -2 \end{pmatrix} \Rightarrow X = A^{-1} \cdot B$

$\det A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 2 + 1 = 3 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = (-1)^2 \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} = 0$

$A_{12} = (-1)^3 \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$

$A_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = 1$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 2$

$A_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -2$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$

$A^{-1} = \frac{1}{3} \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & -2 \\ -3 & 2 & -1 \end{pmatrix}^T$

$A^{-1} = \begin{pmatrix} 0 & 1/3 & 1/3 \\ 0 & -1/3 & -2/3 \\ -1 & 2/3 & 1/3 \end{pmatrix}$

$X = \begin{pmatrix} 0 & 1/3 & 1/3 \\ 0 & 1/3 & -2/3 \\ -1 & 2/3 & -1/3 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ -1 & -2 \end{pmatrix}$

$X = \begin{pmatrix} 4/3 - 1/3 & 5/3 - 2/3 \\ 4/3 + 2/3 & 5/3 + 4/3 \\ -2 + 8/3 + 1/3 & -3 + 10/3 + 2/3 \end{pmatrix}$

$X = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{pmatrix}$

② 1) $\begin{cases} x_1 + 2x_2 + 2x_3 = 2 \\ 2x_1 + 3x_2 + 5x_3 = 7 \\ 2x_1 + 7x_2 + 4x_3 = -2 \end{cases}$

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 5 \\ 2 & 7 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix}$$

$\Delta = 12 + 20 + 28 + 12 - 35 - 16 = -13 \neq 0$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 2 \\ 7 & 3 & 5 \\ -2 & 7 & 4 \end{vmatrix} = 24 + 98 - 20 + 12 - 70 - 56 = 18$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 7 & 5 \\ 2 & -2 & 4 \end{vmatrix} = 28 - 8 + 20 - 28 + 10 - 16 = 6$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 7 \\ 2 & 7 & -2 \end{vmatrix} = -6 + 28 + 28 - 12 - 49 + 8 = -3$$

$x_1 = \frac{18}{-13}$ $\quad x_2 = \frac{6}{-13}$ $\quad x_3 = \frac{3}{13}$ | Отг: $\left(-\frac{18}{13}; -\frac{6}{13}; \frac{3}{13}\right)$

2) $\begin{cases} x_1 + 2x_2 + 3x_3 = 17 \\ 2x_1 - x_2 + x_3 = -28 \\ -x_1 + 6x_2 + 5x_3 = 39 \end{cases}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ -1 & 6 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 17 \\ -28 \\ 39 \end{pmatrix}$$

$\Delta = -5 + 36 - 2 - 3 - 6 - 20 = 36 - 36 = 0 \Rightarrow$ H.P.